Recent Developments on Exact Solvers for the (Prize-Collecting) Steiner Tree Problem

In this tutorial we study the Steiner tree problem (STP) in graphs and its more general version, known as the asymmetric prize-collecting Steiner tree problem (APCSTP). Several well-known network design problems can be transformed to the APCSTP, including the Steiner tree problem (STP), prize-collecting Steiner tree problem (PCSTP), maximum-weight connected subgraph problem (MWCS), and the node-weighted Steiner tree problem (NWSTP).

In the first part of this tutorial we will recapitulate some of the most frequently used MIP formulations for the (prize-collecting) STP, and then we will present details of our implementation which won the DIMACS challenge on Steiner Trees in 2014 in many categories. Our approach is based on the idea of "thinning out" the usual models for the sake of getting a more agile framework. In particular, our model involves node variables only, which is rather appealing for the "uniform" cases where all edges have the same cost. We will also demonstrate how to build a unified solver on top of the new node-based model and the previous state-of-the-art model (defined in the space of arc and node variables). The solver relies on local branching, initialization heuristics, preprocessing and local search procedures. A filtering mechanism is applied to automatically select the best algorithmic ingredients for each instance individually.

In the second part of the talk we will focus on the more recent developments concerning an exact branch-and-bound solver for the APCSTP. The main component of our framework is a new dual ascent algorithm for the rooted APCSTP, which generalizes Wong's dual ascent algorithm for the Steiner arborescence problem. The lower bounds and dual information obtained from the algorithm are exploited within powerful bound-based reduction tests and for guiding primal heuristics. The framework is complemented by additional alternative-based reduction tests. Extensive computational results on benchmark instances for the PCSTP, MWCS, and NWSTP indicate the framework's effectiveness, as most instances from literature are solved to optimality within seconds, including most of the (previously unsolved) largest instances from the recent DIMACS Challenge on Steiner trees.
The latter is an open-source project (available on GitHub) which we hope will help to boost the further development of exact approaches for this challenging class of network design problems.